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NASA Technical Memorandum 87823

Concatenated Coding Scheme for Telecommand Error Control

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1985

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Introduction

A new telecommand standard ~~under preparation~~ by CCSDS defines new procedures, protocols and data structures for command transmission from ground source to spacecraft data handling subsystem.[1], [2] This standard specifies the need for a concatenated coding scheme for error control which is accomplished using error detection and retransmission, called automatic repeat request (ARQ).[3] In an ARQ system, when errors are detected at the receiver, a request is sent for the transmitter to repeat the message and this continues until the message is received correctly. The primary performance requirements for telecommand are expressed by two system level parameters. These are the probability of a message eraser because of a detected error, and the probability of there being an undetected message error in an accepted message. In the new standard the probability of undetected message error is stringent which results from the philosophy that when a command passes the acceptance tests it is 'error free' and does not require confirmation by echoing commands through the telemetry channel to the ground terminal for correlation before acceptance.

It is NASA's position that only the inner code of the proposed concatenated scheme is required to meet the primary performance requirements for telecommanding and any additional protection of the message by using of an outer code be optional.

Concatenated Code Scheme

A concatenated coding scheme for error control is shown in Figure 1. Linear codes are used for both the inner block code C_b, $(n_b k_b)$ and the outer frame code C_f (n_f, k_f). The encoding is accomplished in two stages. A message of k_f bits is first encoded into a frame code word of n_f bits in the outer encoder. Then the n_f bits is divided into m k_b bit segments. Each segment is then encoded into a code word of n_b bits in the inner encoder. The frame format segmented on block code words is shown in Figure 2.

Throughout this study the inner code is a distance -4 Hamming code with generator polynomial

$$g(x) = X^7 + X^6 + X^2 = 1$$

with a maximum length of n_b of 63 bits. Two decoding algorithms using this inner code are considered.

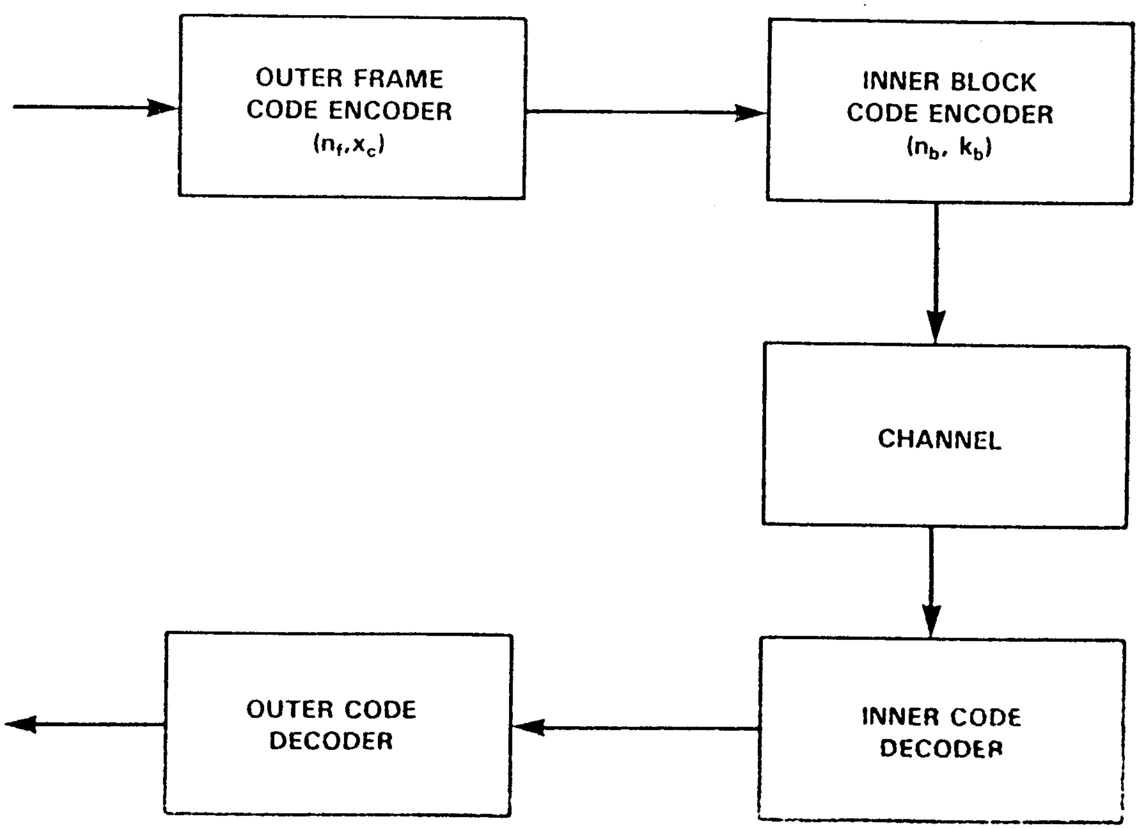
In the first case when a code word is received, it is decoded based on C_b as a single bit error corrector and double error detector. After decoding the parity bits are removed, k_b bits are buffered. If there are more than 2 errors in a received block the decoded segment may contain undetectable errors. After m blocks of a frame have been decoded the buffer contains m_f decoded bits (mk_b bits). Decoding is performed on these m_f bits according to the outer code C_f.

In the second case when a code word is received, it is decoded based on C_b as a triple error detector. After decoding the parity bits are removed, k_b bits are buffered. If there are more than 3 errors in a received block, the decoded segment may contain undetected errors. After m blocks of a frame have been decoded, the buffer contains n_f decoded bits and these bits are then decoded according to the outer code C_f.

In each of the above cases the outer decoder is used for error detection on the n_f bits according to C_f. If no error is detected the k_f information bits are assumed to be 'error free' and are accepted. If the presence of errors is detected, the n_f bits are erased and retransmission of the frame is requested. The outer code is a distance -4 shortened Hamming code with generator polynomial

$$g(x) = X^{16} + X^{12} + X^5 + 1$$

This code is the X.25 standard for packet switched networks. The natural length of this code is 32,767 bits, which is restricted by formatting to a maximum of 2048 bits (256 octets).



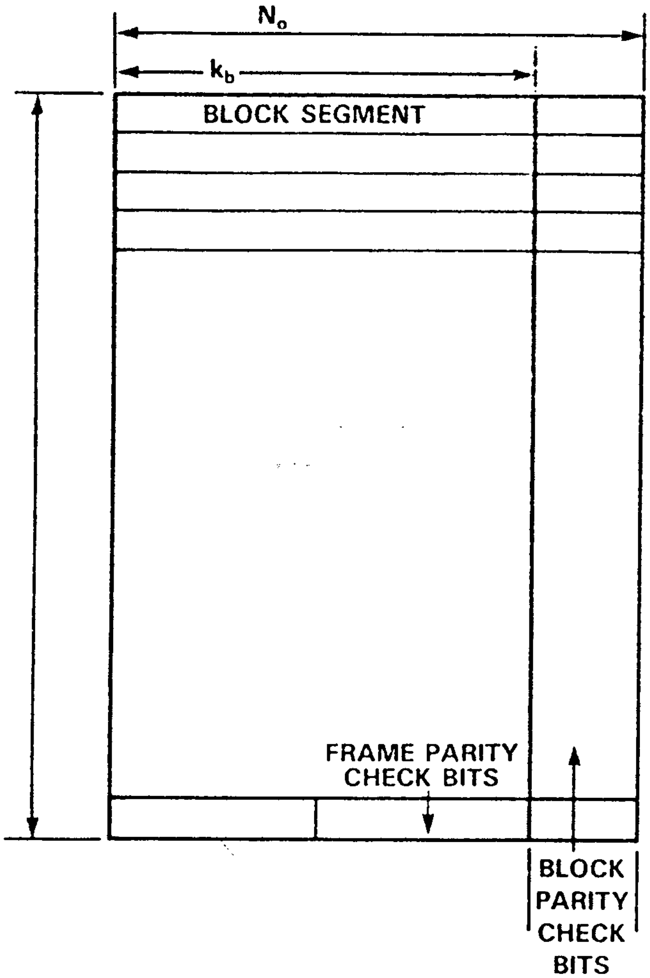


Figure 2. Frame Format.

Performance Requirements

The primary performance requirements for telecommand channel are expressed by specifying two system level parameters with the CCSDS transfer frame layer: these are the probability of a frame being erased because of a detected error shall occur less than 1 per 10^3 frames that are transmitted, and the probability of there being an undetected error in an accepted frame shall be less than one undetected frame error per 10^9 frames. Both of these parameters assume a binary system channel when the command channel is operating above a threshold of one error in 10^5 bits.

Figure 3 shows the relationship between the number of blocks per frame (m) and the total number of frames transmitted per 5 year expected mission life as a function of transmitted bit rate. This family of curves (solid lines) assume telecommanding for 10 minutes per orbit with 64 bits per blocks and a 90 minute orbit. The bit rate of 2K b/s (G STND) and 1 K b/s (TDRSS) are the maximum rate for the second generation standard transponder presently in development by NASA for near earth orbiting missions. The dotted curve assumes telecommanding continues 24 hours per day for 5 years at a deep space rate of 32 bits/second. The 32 bit/second rate was used at Jupiter distance by NASA. Both Space Telescope and Earth Radiation Budget Satellites will command at 1 K b/s rate and expect to transmit less than 2×10^8 commands (m=1) for their mission life.

The data depicted in the family of curves shown in Figure 3 indicates the requirement of one error in 10^9 frames satisfies the 'error free' philosophy of CCSDS for both near earth and deep space missions.

Analytical Results

Results were calculated for the probability of undetected frame error and the channel efficiencies for both the concatenated code scheme and for these systems utilizing just the inner block code. These calculations assume a binary symmetric channel and the system threshold occurs at a channel bit error rate 1 error in 10^5 bits.

The probability of undetected frame error (P_{uf}) was calculated for the inner code as a single error corrector/double error detector (SEC/DED) using

$$P_{mf} = \sum_{h=1}^{m} \binom{m}{h} [p_{pw}]^h \, [1 - p_{wb}]^{m-h}$$

where

$$P_{wb} = 4 \, A_4 \, \epsilon^3 \, (1-\epsilon)^{n-3}$$

Recall the m is the number of segments in a frame, n is the code length, A_4 is the number of code words with the weight 4, [3) and ϵ, is the channel bit error rate. The curve labeled SEC/DEC shown in Figure 4 is obtained from the previous equation with $n = 64$, $A_4 = 9765$ and $\epsilon = 10^{-5}$.

Also shown in Figure 4 is a curve when the decoder is configured as a Triple Error Detector (TED). The same equation P_{uf} is used for this curve with:

$$P_{wb} = A_4 \, \epsilon^4 \, (1-\epsilon)^{n-4}$$

with $n=64$, $A_4=9765$ and $\epsilon=10^{-5}$.

Upper and lower bounds were derived on the probability of undetected error for a concatenated scheme where the inner code is used for SEC/DED and the outer code is used only for error detection, see Attachment A for equations and analysis. Results showing the upper bound is plotted in Figure 5 assuming a channel bit error rate of 10^{-5} and the inner code block length is fixed at 63 bits.

Channel efficiencies (η) were derived using Weldon's ARQ strategy [4], [5] for both coding schemes. The derivation considers parameters for both near earth and deep space

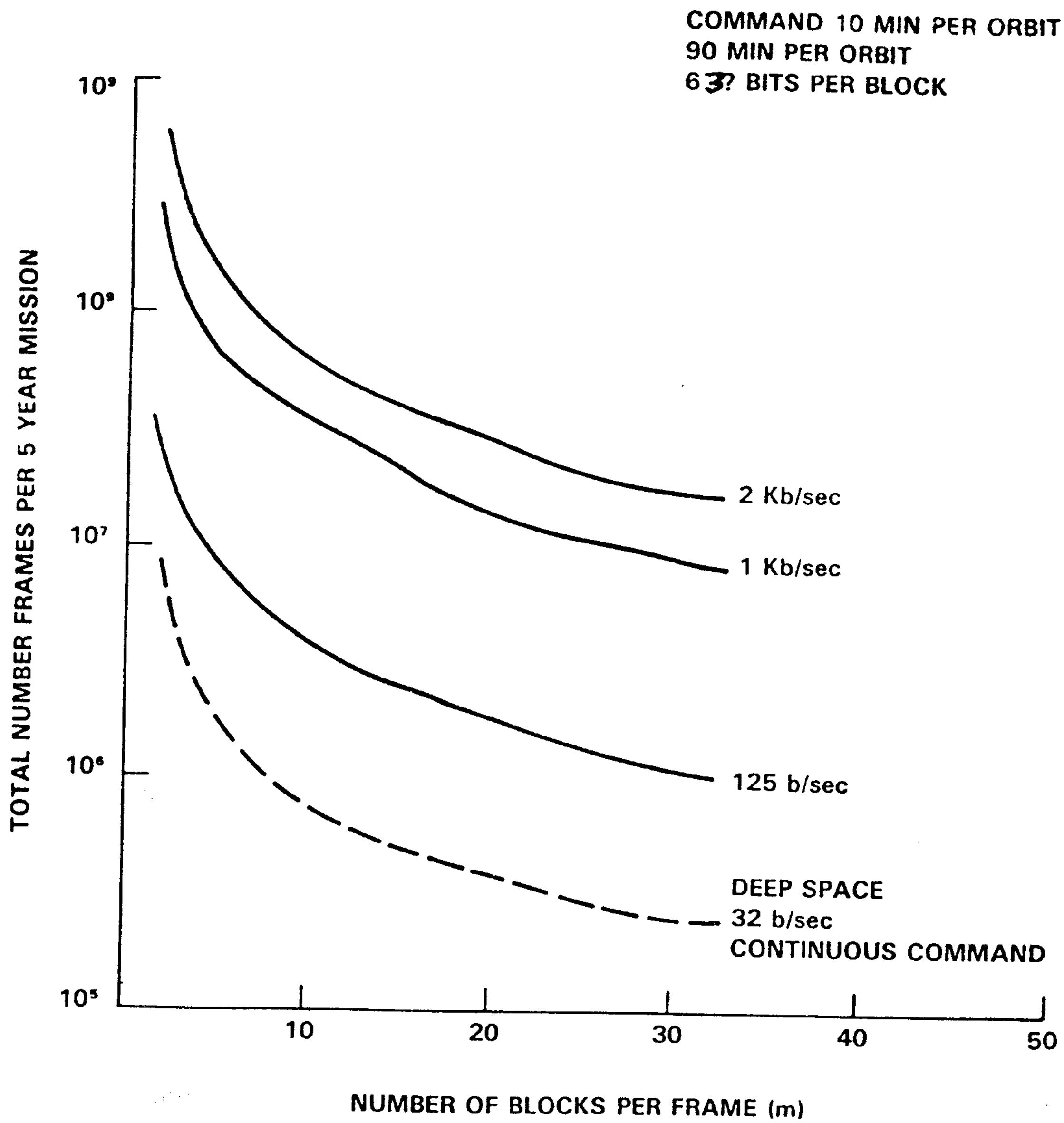


Figure 3. Total number of frames per 5 year mission for various command data rates.

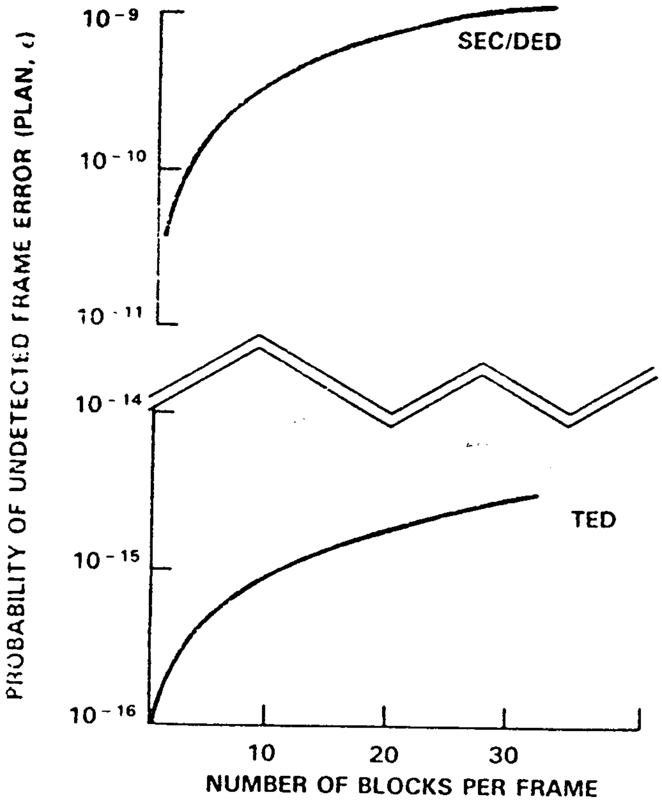


Figure 4. Probability of undetected frame error for inner distance 4 Hamming Code (65.56), channel error rate 10^{-5}

mission, and for different Command Operation Procedures (COP) [2]

COP1A & 4A sequential acceptance and retransmission
COP2A sequence independent acceptance with selective retransmission

$$\eta = \frac{p'}{1 + (a-1)(1-p)^{q+1}} \cdot \frac{k}{n}$$

where

$$a = \left[3 + 2\,\frac{Rd}{nc} \right]$$

$$q = \begin{cases} O & \text{for COP1 \& 4} \\ \left[\dfrac{B}{an} \right] & \text{for COP2} \end{cases}$$

p = probability of no error
d = communication distance
c = speed of light
R = channel bit rate
B = storage capacity at receiver
k = number of information bits per frame
n = number of bits per frame

Throughput efficiency results based on the above equations are shown in Tables 1 through 4 for both coding schemes, for near earth parameters (d = 100 miles, R = 1Kbps) and deep space parameters ($d = 500 \times 10^6$ miles, R = 32)

Table 1: Throughput efficiency for (63,56) distance 4 Hamming Code, TED with m=32 (k/n=.8750).

	COP/STORAGE SIZE			
BER	COP1/4	COP2		
		1K Bits	10K Bits	100K Bits
Near Earth				
10^{-4}	.524(401)	.524(401)	.671(233)	.715(183)
10^{-5}	.839(41)	.839(41)	.857(21)	.858(19)
10^{-6}	.870(6)	.870(6)	.873(2)	.873(2)
Deep Space				
10^{-4}	.043(950)	.043(950)	.043(950)	.043(950)
10^{-5}	.316(639)	.316(639)	.316(639)	.316(639)
10^{-6}	.744(150)	.744(150)	.744(150)	.744(150)

Note ( ) is number of rejects per 1000 transmissions

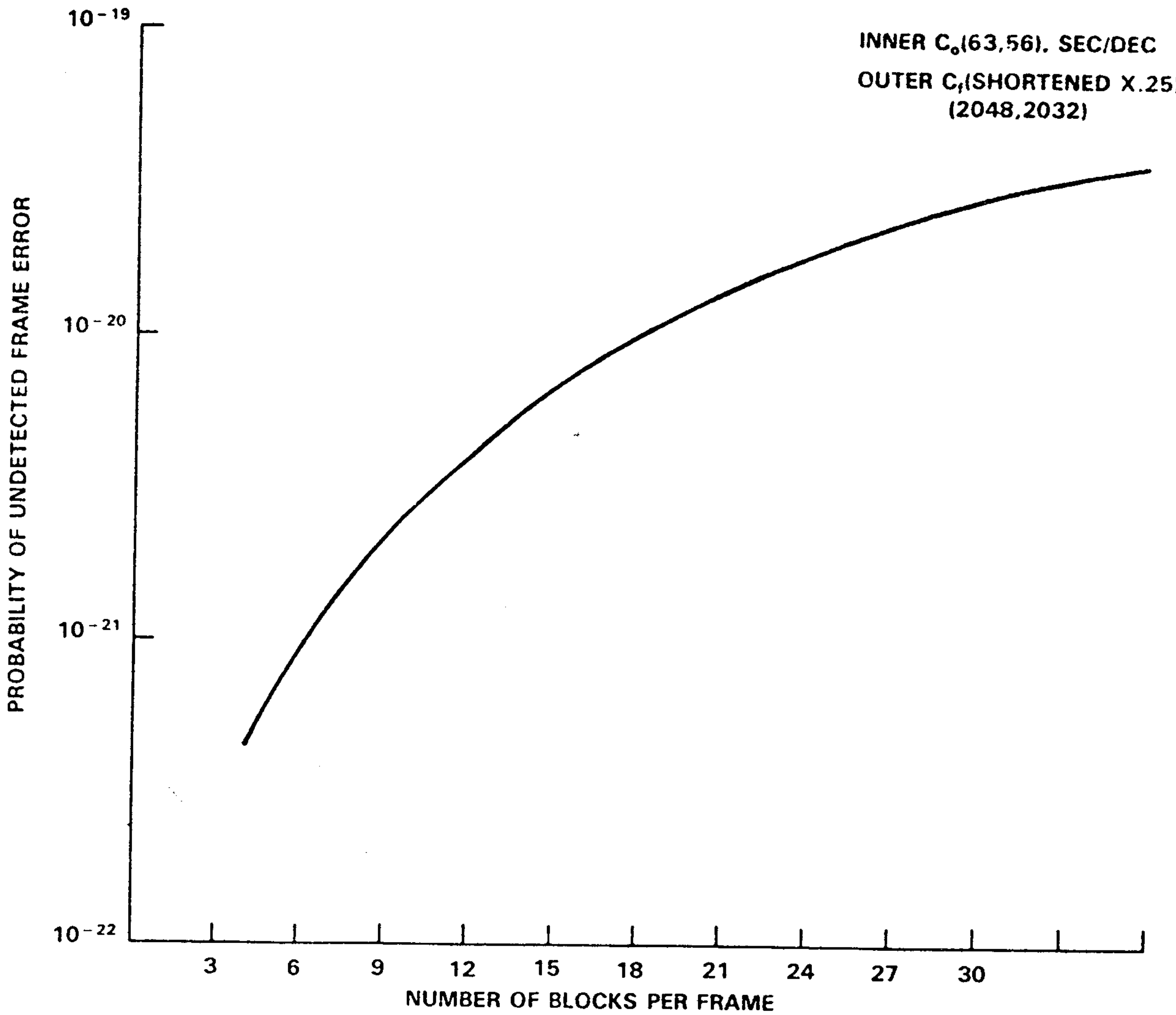


Figure 5. Upper Bound on the probability of undetected error for concatenated coding scheme, channel bit error rate 10^{-5}.

Table 2: Throughput efficiency for (63,56) distance 4 Hamming Code, SEC/DED with m=32 (k/m=.8750).

BER	COP/STORAGE SIZE			
	COP1/4		COP2	
Near Earth		1K Bits	10K Bits	100K Bits
10^{-4}	.873(2)	.873(2)	.875(0)	.875(0)
10^{-5}	.875(0)	.875(0)	.875(0)	.875(0)
10^{-6}	.875(0)	.875(0)	.875(0)	.875(0)
Deep Space				
10^{-4}	.830(51)	.830(51)	.830(51)	.830(51)
10^{-5}	.875(0)	.875(0)	.875(0)	.875(0)
10^{-6}	.875(0)	.875(0)	.875(0)	.875(0)

Note ( ) is number of rejects per 1000 transmissions

Table 3: Throughput efficiency for (63.56) distance 4 Hamming Code, SEC/DED inner code concatenated with a shortened X.25 Code, error detect only with m=32 (k/n=1776/2048=.8672).

	COP/STORAGE SIZE			
BER	COP1/4		COP2	
Near Earth		1K Bits	10K Bits	100K Bits
10^{-4}	.865(3)	.865(3)	.867(0)	.867(0)
10^{-5}	.867(0)	.867(0)	.867(0)	.867(0)
10^{-6}	.867(0)	.867(0)	.867(0)	.867(0)
Deep Space				
10^{-4}	.823(51)	.823(51)	.823(51)	.823(51)
10^{-5}	.867(0)	.867(0)	.867(0)	.867(0)
10^{-6}	.867(0)	.867(0)	.867(0)	.867(0)

Note ( ) is number of rejects per 1000 transmissions

Table 4: Throughput efficiency for (63,56) distance 4 Hamming Code, TED with m=1 (k/n=.8750).

	COP/STORAGE SIZE			
BER	COP1/4		COP2	
Near Earth		1K Bits	10K Bits	100K Bits
10^{-4}	.8590(18)	.875(0)	.875(0)	.875(0)
10^{-5}	.8733(2)	.875(0)	.875(0)	.875(0)
10^{-6}	.8748(0)	.875(0)	.875(0)	.875(0)

Note ( ) is number of rejects per 1000 transmissions

Recommendation

NASA recommends that only the inner (63.56) Hamming Code be required for telecommand protection. The data shown in Figure 4 and Table 2 indicates that when this code operates as a SEC/DED the system meets the CCSDS requirement of undetected frame error and for throughput efficiency when operating above threshold.

If additional protection beyond one frame error in 10^9 frames is required either the inner (63.56) Hamming Code operating as a TED with reduced throughput efficiency; or the inner code concatenated with the shortened X.25 outer code should be recommended. The concatenated coding scheme will provide a probability of undetected frame error of less than 10^{-19} with excellent efficiency see Figure 5 and Table 3.

Reference

[1] "CCSDS Telecommand Channel Coding and Procedures", Issue-0, May 1984.

[2] "CCSDS Telecommand Transfer Frame Formats and Procedures", Issue-3, July 1984.

[3] S. Lin, D. Costello, "Error Control Coding: Fundamentals and Applications", Prentice Hall, 1983.

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NASA

Report Documentation Page

1. Report No. NASA TM 87823	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle: Concatenated Coding Scheme for Telecommand Error Control		5. Report Date: April 1987
		6. Performing Organization Code: 730
7. Author(s): Warner H. Miller and James C. Morakis		8. Performing Organization Report No.: 87B0315
		10. Work Unit No.
9. Performing Organization Name and Address: Goddard Space Flight Center, Greenbelt, Maryland 20771		11. Contract or Grant No.
		13. Type of Report and Period Covered: Technical Memorandum
12. Sponsoring Agency Name and Address: National Aeronautics and Space Administration, Washington, D.C. 20546-0001		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

A concatenated coding scheme for error control in a telecommunication channel is analyzed. The inner code is used either for both error correction and detection, or just error detection, the outer code is used only for error detection. A retransmission is requested if the outer code detects the presence of errors after the inner code decoding. The probability of undetected error and throughput efficiency is derived for the concatenated coding scheme and is compared to the performance of having just the inner coding scheme.

17. Key Words (Suggested by Author(s))	18. Distribution Statement: Unclassified—Unlimited Subject Category

19. Security Classif. (of this report): Unclassified	20. Security Classif. (of this page): Unclassified	21. No. of pages	22. Price